UNIVERSITY OF CAMBRIDGE

# ECONOMICS
# ADMISSIONS ASSESSMENT

## Specimen Paper – New Format for 2020 60 minutes

## SECTION 1

### INSTRUCTIONS TO CANDIDATES

**Please read these instructions carefully, but do not open this question paper until you are told that you may do so.** This paper is Section 1 of 2.

A separate answer sheet is provided for this paper. Please check you have one. You also require a soft pencil and an eraser.

Please complete the answer sheet with your candidate number, centre number, date of birth, and name.

At the end of 60 minutes, your supervisor will collect this question paper and answer sheet before giving out Section 2.

This paper contains **two** parts: **A** and **B** and you should attempt **both** parts.

**Part A** Mathematics (20 questions)
**Part B** Advanced Mathematics (20 questions)

You are **strongly** advised to divide your time equally between the two parts: 30 minutes on **Part A** and 30 minutes on **Part B**. The scores for Part A and Part B are reported separately.

This paper contains 40 multiple-choice questions. There are no penalties for incorrect responses, only marks for correct answers, so you should attempt **all** 40 questions. Each question is worth one mark.

For each question, choose the **one** option you consider correct and record your choice on the separate answer sheet. If you make a mistake, erase thoroughly and try again.

You **must** complete the answer sheet within the time limit.

You can use the question paper for rough working, but **no extra paper** is allowed. Only your responses on the answer sheet will be marked.

Dictionaries and calculators are NOT permitted.

**Please wait to be told you may begin before turning this page.**

*This question paper consists of 28 printed pages and 4 blank pages.*



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**Paper content**

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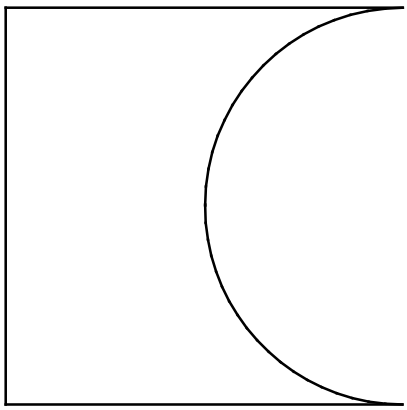
# PART A Mathematics

PART A Mathematics

**1** A square piece of metal has a semicircular piece cut out of it as shown. The area of the remaining metal is $100\,\text{cm}^2$.



[diagram not to scale]

Which one of the following is a correct expression for the length of the side of the square in centimetres?

**A** $20\sqrt{\dfrac{2}{8-\pi}}$

**B** $10\sqrt{\dfrac{2}{4-\pi}}$

**C** $20\sqrt{\dfrac{2}{8+\pi}}$

**D** $10\sqrt{\dfrac{1}{8-\pi}}$

**E** $20\sqrt{\dfrac{1}{4-\pi}}$

**2** In a right-angled triangle PQR the hypotenuse is the side PR.

The length of side PQ is 20 cm and the ratio RQ : PQ is 1 : 2

What is the length of the perpendicular from the hypotenuse to the point Q?

**A** $8\sqrt{5}$ cm

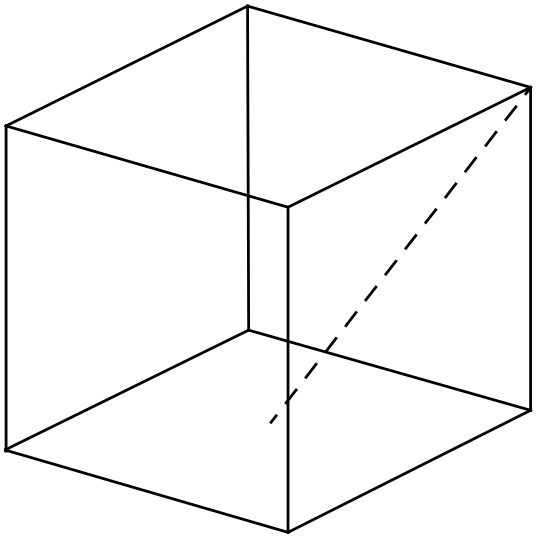
**B** $10\sqrt{2}$ cm

**C** $2\sqrt{5}$ cm

**D** $5\sqrt{2}$ cm

**E** $4\sqrt{5}$ cm

**3** A cube has sides of unit length. What is the length of a line joining a vertex to the midpoint of one of the opposite faces (the dashed line in the diagram below)?



[diagram not to scale]

**A** $\sqrt{\frac{3}{2}}$

**B** $\sqrt{2}$

**C** $\sqrt{\frac{5}{2}}$

**D** $\sqrt{3}$

**E** $\sqrt{5}$

**4** If you look at a clock and the time is 9:45, what is the angle between the hour and the minute hands?

**A** 0°

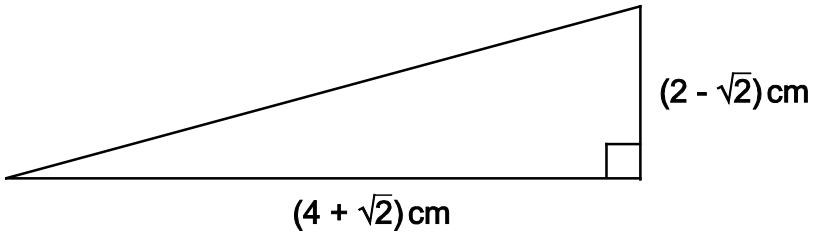
**B** 7.5°

**C** 15°

**D** 22.5°

**E** 30°

**5** The right-angled triangle shown has horizontal and vertical sides measuring $(4+\sqrt{2})$ cm and $(2-\sqrt{2})$ cm respectively.




Calculate the area of the triangle.

**A** $(5+3\sqrt{2})\,\text{cm}^2$

**B** $(3-\sqrt{2})\,\text{cm}^2$

**C** $(3+3\sqrt{2})\,\text{cm}^2$

**D** $(5-\sqrt{2})\,\text{cm}^2$

**6** A solid sphere of radius $r$ fits inside a hollow cylinder. The cylinder has the same internal diameter and length as the diameter of the sphere.

The volume of a sphere is $\frac{4}{3}\pi r^3$, where $r$ is the radius of the sphere.

What fraction of the space inside the cylinder is taken up by the sphere?

**A** $\frac{1}{4}$

**B** $\frac{1}{3}$

**C** $\frac{1}{2}$

**D** $\frac{2}{3}$

**E** $\frac{3}{4}$

**7** Which of the expressions below has the largest value for $0 < x < 1$?

**A** $\frac{1}{x}$

**B** $x^2$

**C** $\frac{1}{(1+x)}$

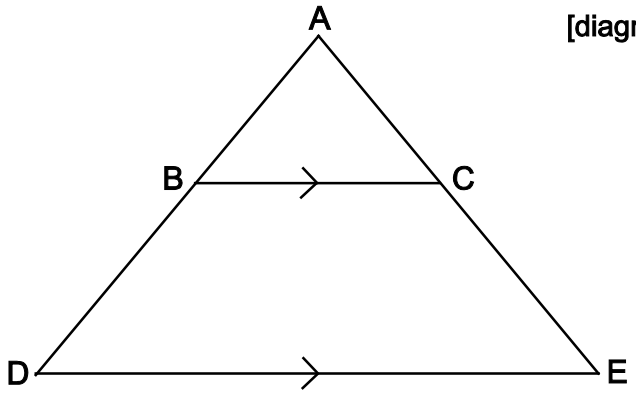
**D** $\frac{1}{\sqrt{x}}$

**E** $\sqrt{x}$

**8** A shape is formed by drawing a triangle ABC inside the triangle ADE.

BC is parallel to DE.

AB = 4 cm  BC = $x$ cm  DE = $x + 3$ cm  DB = $x - 4$ cm




[diagram not to scale]

Calculate the length of DE.

**A** 5 cm

**B** 7 cm

**C** 9 cm

**D** $4 + 2\sqrt{7}$ cm

**E** $7 + 2\sqrt{7}$ cm

**9** Two variables are connected by the relation: $P \propto \frac{1}{Q^2}$

$Q$ is increased by 40%.

To the nearest percent, describe the change in $P$ in percentage terms.

**A** 29% decrease

**B** 44% decrease

**C** 49% decrease

**D** 51% decrease

**E** 80% decrease

**F** 96% decrease

**10** Three variables $x$, $y$ and $z$ are known to be related to each other in the following ways:

$x$ is directly proportional to the square of $z$.
$y$ is inversely proportional to the cube of $z$.

Which of the following correctly describes the relationship between $x$ and $y$?

**A** The square of $x$ is directly proportional to the cube of $y$.

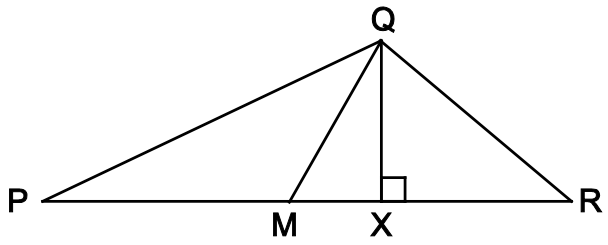
**B** The square of $x$ is inversely proportional to the cube of $y$.

**C** The cube of $x$ is directly proportional to the square of $y$.

**D** The cube of $x$ is inversely proportional to the square of $y$.

**E** $x$ is directly proportional to $y^6$.

**11** In the triangle PQR shown below:

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X lies on PR

$\angle$QXR is 90°

$\frac{QX}{PX} = \frac{1}{6}$

$\frac{QX}{XR} = \frac{2}{3}$

M is the midpoint of PR.

What is $\frac{QX}{MX}$?

**A** $\frac{1}{9}$

**B** $\frac{5}{12}$

**C** $\frac{4}{9}$

**D** $\frac{1}{2}$

**E** $\frac{5}{6}$

**12** Solve the inequality $x^2 \geq 8 - 2x$

**A** $x \geq 4$

**B** $x \leq 2$ and $x \geq -4$

**C** $x \geq -2$ and $x \leq 4$

**D** $x \geq 2$ or $x \leq -4$

**13** The total surface area of a cylinder, measured in square centimetres, is numerically the same as its volume, measured in cubic centimetres.

The radius of the cylinder is $r$ cm, the height is $h$ cm.

Express $h$ in terms of $r$.

**A** $h = \dfrac{2r}{r-2}$

**B** $h = \dfrac{2r}{r+2}$

**C** $h = r + 2$

**D** $h = r - 2$

**E** $h = 2r(r-2)$

**14** How many different integers, $n$, are there such that the difference between $2\sqrt{n}$ and 7 is less than 1?

**A** 0

**B** 2

**C** 4

**D** 6

**E** 8

**15** The square PQRS is positioned so that its vertices are at the points with coordinates: (1, 1), (–1, 1), (–1, –1) and (1, –1).

The square is rotated clockwise through 90° about the origin and then reflected in the line $y = x$

Which transformation will return the square to its original orientation?

**A** A reflection in the $x$-axis.

**B** A reflection in the $y$-axis.

**C** A reflection in the line $y = -x$

**D** A rotation of 90° clockwise about the origin.

**E** A rotation of 90° anticlockwise about the origin.

**16** To get to work, Sylvie first catches a bus and then catches a train.

The probability that the bus is on time is 0.6

The probability that the bus is late is 0.4

If the bus is on time, then the probability that she will catch the train is 0.8

If the bus is late, then the probability that she will catch the train is 0.6

Given that Sylvie catches the train, what is the probability that the bus was on time?

**A** $\frac{1}{3}$

**B** $\frac{12}{25}$

**C** $\frac{2}{5}$

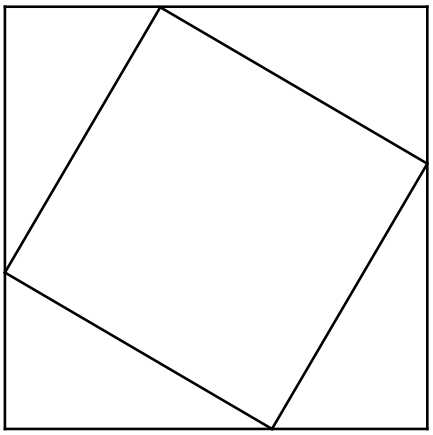
**D** $\frac{3}{5}$

**E** $\frac{2}{3}$

**F** $\frac{18}{25}$

**G** $\frac{6}{7}$

**17** A design is set up by joining the points which are one third of the way along the sides of a square. This forms a second square as shown.



[diagram not to scale]

This process is repeated.

Calculate the area of the fourth square as a fraction of the original square.

**A** $\frac{1}{27}$

**B** $\frac{5\sqrt{5}}{81}$

**C** $\frac{125}{729}$

**D** $\frac{25}{81}$

**E** $\frac{5\sqrt{5}}{27}$

**18** The equation connects the variables $M$, $x$, $y$, $z$, $P$ and $Q$.

$$M = \frac{(x+y)^2 z}{P} Q$$

The following changes are made:

$x$ and $y$ are both increased by 50%

$z$ is decreased by 20%

$P$ is doubled and $Q$ remains the same.

What is the resulting percentage change in $M$?

**A** 2.5% decrease

**B** 2.5% increase

**C** 10% decrease

**D** 10% increase

**E** 20% decrease

**F** 20% increase

**19** I have two six-sided dice, each with faces numbered from 1 to 6. One of the dice is fair, but the other is not; it will land on numbers 1 to 5 with equal probability, but lands on 6 with a different probability.

When I roll the dice the probability that I get a total of 12 is $\frac{1}{18}$.

What is the probability that I get a total of 2 when I roll the dice?

**A** $\frac{1}{72}$

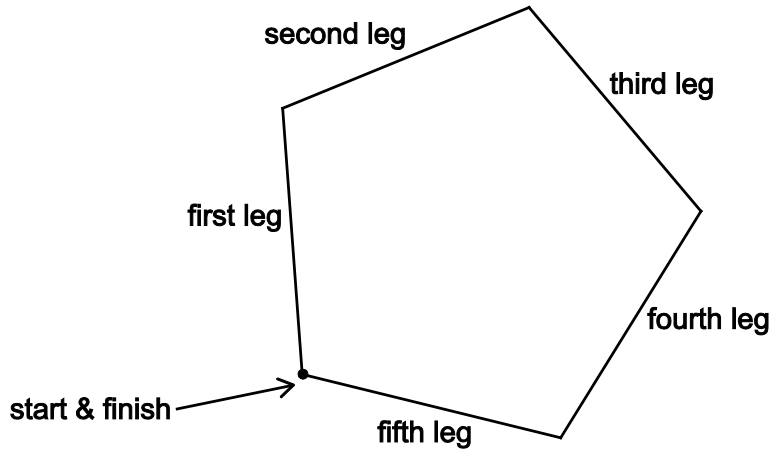
**B** $\frac{1}{45}$

**C** $\frac{1}{36}$

**D** $\frac{1}{18}$

**E** $\frac{1}{9}$

**20** A cross-country running track is in the shape of a regular pentagon.

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Competitors run clockwise around the track.

When on the third leg of the course they run on a bearing of 110°.

What bearing do they run on for the first leg?

**A** 034°

**B** 038°

**C** 106°

**D** 178°

**E** 182°

**F** 244°

**G** 322°

**H** 326°

# PART B Advanced Mathematics

**21** Given that $a^x b^{2x} c^{3x} = 2$, where $a$, $b$ and $c$ are positive real numbers, then $x =$

**A** $\log_{10}\left(\frac{2}{a+2b+3c}\right)$

**B** $\frac{\log_{10} 2}{\log_{10}(a+2b+3c)}$

**C** $\frac{2}{\log_{10}(a+2b+3c)}$

**D** $\frac{2}{a+2b+3c}$

**E** $\log_{10}\left(\frac{2}{ab^2c^3}\right)$

**F** $\frac{\log_{10} 2}{\log_{10}(ab^2c^3)}$

**G** $\frac{2}{\log_{10}(ab^2c^3)}$

**H** $\frac{2}{ab^2c^3}$

**22** Which one of the following is a simplification of $\frac{x^2-4}{x^2-2x}$ where $x \neq 2$ and $x \neq 0$?

**A** $\frac{x-4}{x-2}$

**B** $\frac{x-2}{x}$

**C** $\frac{2}{x}$

**D** $\frac{x+2}{x}$

**E** $\frac{x+2}{x+1}$

**23** Which one of the following numbers is largest in value?

(All angles are given in radians.)

**A** $\tan\left(\frac{3\pi}{4}\right)$

**B** $\log_{10} 100$

**C** $\sin^{10}\left(\frac{\pi}{2}\right)$

**D** $\log_2 10$

**E** $\left(\sqrt{2}-1\right)^{10}$

**24** When $x = 2$ is substituted in the expression $x^3 + px^2 + qx + p^2$ the result is 0

When $x = 1$ is substituted into the same expression, the result is $-3.5$

Find all possible values of $p$.

**A** $p = -1 \pm \frac{\sqrt{6}}{3}$

**B** $p = 1$ or $p = -3$

**C** $p = 1$

**D** $p = 1 \pm \sqrt{7}$

**E** there are no values for $p$

**25** The sum of the roots of the equation $2^{2x} - 8 \times 2^{x} + 15 = 0$ is

**A** 3

**B** 8

**C** $2\log_{10} 2$

**D** $\log_{10}\left(\frac{15}{4}\right)$

**E** $\frac{\log_{10} 15}{\log_{10} 2}$

**26** A square *PQRS* is drawn above the $x$-axis with the side *PQ* on the $x$-axis.

*P* is the point $(-5, 0)$ and Q is the point $(1, 0)$.

A circle is drawn inside the square with diameter equal in length to the side of the square.

Which one of the following is an equation of the circle?

**A** $x^2 + y^2 - 4x + 6y + 4 = 0$

**B** $x^2 + y^2 - 4x + 6y + 9 = 0$

**C** $x^2 + y^2 + 4x - 6y + 4 = 0$

**D** $x^2 + y^2 + 4x - 6y + 9 = 0$

**E** $x^2 + y^2 - 6x - 4y + 9 = 0$

**F** $x^2 + y^2 - 6x + 4y + 4 = 0$

**G** $x^2 + y^2 + 6x - 4y + 4 = 0$

**H** $x^2 + y^2 + 6x + 4y + 9 = 0$

**27** For any real numbers $a$, $b$ and $c$ where $a \geq b$, consider these three statements:

**1** $-b \geq -a$

**2** $a^2 + b^2 \geq 2ab$

**3** $ac \geq bc$

Which of the above statements must be true?

**A** none of them

**B** 1 only

**C** 2 only

**D** 3 only

**E** 1 and 2 only

**F** 1 and 3 only

**G** 2 and 3 only

**H** 1, 2 and 3

**28** The first term of a convergent geometric series is 8

The fifth term is 2

The sixth term is real and positive.

What is the sum to infinity of this series?

(The sum to infinity of a convergent geometric series is given by $\frac{a}{1-r}$, where $a$ is the first term and $r$ is the common ratio.)

**A** $8\left(1+\sqrt{2}\right)$

**B** $8\left(1-\sqrt{2}\right)$

**C** $8\left(2+\sqrt{2}\right)$

**D** $8\left(2-\sqrt{2}\right)$

**E** 16

**F** $\frac{8\sqrt[5]{4}}{\sqrt[5]{4}-1}$

**G** $\frac{8\sqrt[5]{4}}{\sqrt[5]{4}+1}$

**29** The sequence $a_n$ is given by the rule:

$$a_1 = 2$$
$$a_{n+1} = a_n + (-1)^n \text{ for } n \geq 1$$

What is $\sum_{n=1}^{100} a_n$

**A** 150

**B** 250

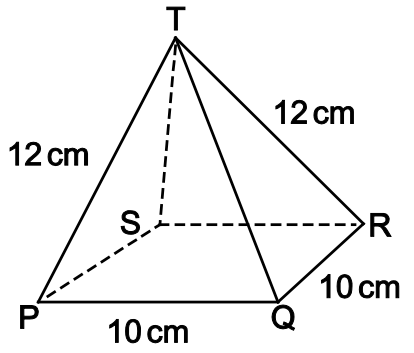
**C** $-4750$

**D** 5150

**E** $4\left(1-\left(\frac{1}{2}\right)^{100}\right)$

**F** $4\left(\left(\frac{3}{2}\right)^{100}-1\right)$

**30** A box is a hollow pyramid. The base of the box is a square with sides 10 cm and all the slant edges of the box are 12 cm long.

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What is the angle made by the slant edge TP with the base PQRS?

**A** $\sin^{-1}\frac{2\sqrt{5}}{12}$

**B** $\sin^{-1}\frac{5}{12}$

**C** $\sin^{-1}\frac{5\sqrt{2}}{12}$

**D** $\cos^{-1}\frac{2\sqrt{5}}{12}$

**E** $\cos^{-1}\frac{5}{12}$

**F** $\cos^{-1}\frac{5\sqrt{2}}{12}$

**31** How many real roots does the equation $x^4 - 4x^3 + 4x^2 - 10 = 0$ have?

**A** 0

**B** 1

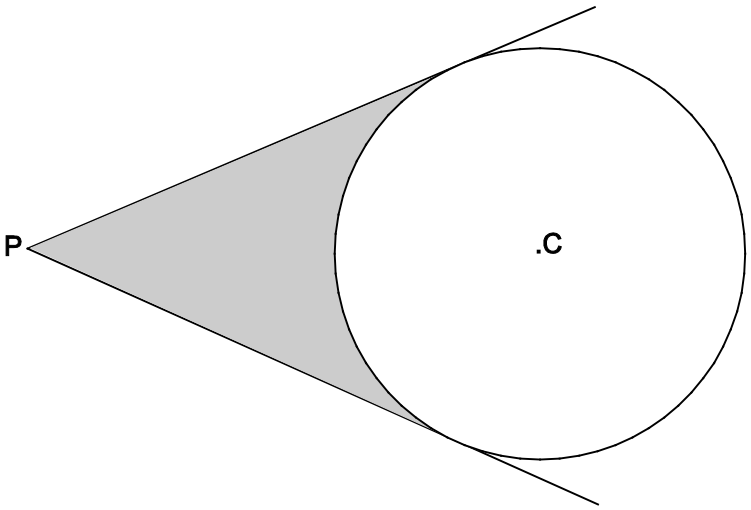
**C** 2

**D** 3

**E** 4

**32** Tangents are drawn from a point *P* to a circle of radius 10 cm.

The centre of the circle is *C* and the distance *PC* is 20 cm.

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Which one of the following is an expression for the shaded area in square centimetres?

**A** $\frac{100}{3}\left(3\sqrt{3}-\pi\right)$

**B** $\frac{100}{3}\left(3\sqrt{5}-\pi\right)$

**C** $\frac{50}{3}\left(6\sqrt{3}-\pi\right)$

**D** $\frac{50}{3}\left(6\sqrt{5}-\pi\right)$

**E** $\frac{50}{3}\left(\sqrt{3}-2\pi\right)$

**F** $\frac{50}{3}\left(2\pi-\sqrt{3}\right)$

**33** Given that $7\cos\theta - 3\tan\theta\sin\theta = 1$, which one of the following is true?

**A** $\cos\theta = -\frac{3}{5}$ or $-\frac{1}{2}$

**B** $\cos\theta = -\frac{3}{5}$ or $\frac{1}{2}$

**C** $\cos\theta = \frac{3}{5}$ or $\frac{1}{2}$

**D** $\cos\theta = \frac{3}{5}$ or $-\frac{1}{2}$

**34** A triangle is to be drawn with sides that are integer lengths in centimetres, and a total perimeter of 12 cm.

How many different (non-congruent) triangles can be drawn?

**A** 1

**B** 2

**C** 3

**D** 10

**E** 12

**35** For what values of the non-zero real number $a$ does the quadratic equation

$$ax^2 + (a-2)x = 2$$

have distinct real roots?

**A** all values of $a$

**B** $a = -2$

**C** $a > -2$

**D** $a \neq -2$

**E** no values of $a$

**36** The complete set of values of $a$ for which the equation $3x^2 = (a+2)x - 3$ has two real distinct roots is:

**A** no values of $a$

**B** $-4\sqrt{2} < a < 4\sqrt{2}$

**C** $a < -4\sqrt{2},\ a > 4\sqrt{2}$

**D** $-4 < a < 8$

**E** $a < -4,\ a > 8$

**F** $-8 < a < 4$

**G** $a < -8,\ a > 4$

**H** all values of $a$

**37** The angle $x$ is measured in radians and is such that $0 \le x \le \pi$.

The total length of any intervals for which $-1 \le \tan x \le 1$ and $\sin 2x \ge 0.5$ is

**A** $\frac{\pi}{12}$

**B** $\frac{\pi}{6}$

**C** $\frac{\pi}{4}$

**D** $\frac{\pi}{3}$

**E** $\frac{5\pi}{12}$

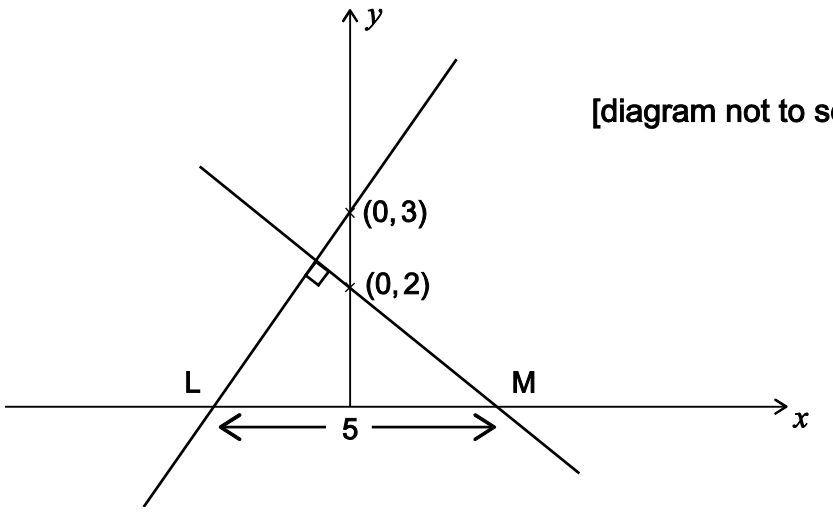
**F** $\frac{\pi}{2}$

**G** $\frac{5\pi}{6}$

**38** The straight line with equation $y = mx + 3$, where m > 0 and m ≠ 1, is perpendicular to the line with equation $y = px + 2$

The lines cut the $x$-axis at the points L and M respectively. The length of LM is 5 units.

What is the value of $m + p$ given that $m > 1$?




**A** $-\frac{8}{3}$

**B** $-\frac{13}{6}$

**C** $-\frac{5}{6}$

**D** $\frac{5}{6}$

**E** $\frac{13}{6}$

**F** $\frac{8}{3}$

**39** The curve $y = x^2$ is translated by the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and then reflected in the line $y = -1$

Which one of the following is an equation of the resulting curve?

**A** $y = -3 - (x - 4)^2$

**B** $y = -3 + (x + 4)^2$

**C** $y = 3 - (x + 4)^2$

**D** $y = 3 + (x - 4)^2$

**E** $y = -5 - (x - 4)^2$

**F** $y = -5 + (x + 4)^2$

**G** $y = 5 - (x + 4)^2$

**H** $y = 5 + (x - 4)^2$

**40** The triangle PQR has a right angle at R.

The length of PQ is 4 cm, correct to the nearest centimetre.

The length of PR is 2 cm, correct to the nearest centimetre.

Find the minimum possible length, in centimetres, of QR.

**A** $\sqrt{6} - \frac{1}{2}$

**B** $2\sqrt{3} - \frac{1}{2}$

**C** $2\sqrt{5} - \frac{1}{2}$

**D** $2\sqrt{5}$

**E** $2\sqrt{3}$

**F** $\sqrt{6}$

**END OF TEST**

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